Game Theory:
Analysis of Championship Style Racing

Jordan Wheeler
Christian Haas, Ph.D.
University of Nebraska Omaha
jmwheeler@unomaha.edu

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Abstract

In the world of Track and Field, "Championship Style Racing" has been an adopted term that many use. A "Championship Style Race" is defined when the field of runners start slow and with a lap or two left, the runners start to pick up pace and sprint to the finish. This adopted strategy has been used more and more, especially when winning is the goal of the race. This paper will go over how Game Theory can be used to determine the optimal strategy for a runner in this type of race. We find a payoff function which uses one of the biggest indicators of a runners ability, $VO2_{max}$.

Contents

1 Introduction and Background

1.1 Championship Track Meets

Competitive championship races in relatively recent years has become more strategic. Rather than getting from start to finish as fast as possible, the focus has shift to being at the finish first, no matter how long it takes to get from start to finish. This tactic is known as a "sit and kick" strategy[1].

A championship meet in track is one which multiple individuals, teams, or countries compete against each other for the top positions, typically first, second, and third (gold, silver, and bronze respectively). Examples of championship meets in track would include the Olympics, which occur every four years in the month of August, World Championships, which occur the years before and after each Olympics, and individual country or continental championships, such as United States of American Track and Field (USATF) Championships and European Championships.

During these championship events, time is typically irrelevant (with a few occasions during country and continental championships where some athletes may still need Olympic standard times, but these are far and few between). Moreover, as the distances increase, time becomes less relevant, which causes individuals who race in distance events (1,500 meters to 10,000 meters) to focus on positions within the field[1] and leg speed, rather than time. Athletes employ different strategies that will increase their chances of a desirable outcome.

The ability to optimally position oneself and choose a best strategy, relative to the rest of the field, has a direct impact on the outcome of the race. Athletes in a championship race often form one pack that runs together in the beginning and middle laps of a race, similar to a peloton in competitive cycling, this is known as the "sit". Towards the end of the race, the runners will position themselves to 'strike' and break away from the pack to get to the finish line first, this is known as the "kick". To fully understand the importance of the final lap(s) of a race, during the 2008 Beijing Olympics, all races, including preliminary heats and finals, from 1,500 meters to 10,000 meters, were decided within the last lap[9].

Professional distance runners train particularly in positioning and leg speed so that they can employ dominate strategies during these championship races. The question becomes what are the dominate strategies and at what point in the race should runners opt to perform a dominate strategy.

[1]When speaking of the all the runners in one event (i.e. the entire group) this is referred as the "field".

1.2 Initial Model

Analysis of championship races will be done through the lens of Game Theory, the initial model will be trivial, but nonetheless, necessary to understanding the Game Theory notation and to form a foundation for the main ideas of championship style racing. Once the initial model is formulated, the complexity will increase as nature and assumptions are introduced. After nature is introduced, assumptions will be made and a payoff function will be created.

1.2.A Players

In any game, there are N players. Let any given player be denoted as Player i, where $i = 1, 2, ..., N$.

For the initial model, consider two players (or runners). Let P_1 be Player 1 and P_2 be Player 2. Both players have the same set of strategies that can be used.

1.2.B Strategies

Let S_i denote the strategy set for Player i. For each strategy set S_i, there is a set of feasible strategies that can be used by Player i, let this be represented by $\{s_i', s_i'', s_i''', ...\}$, where each s represents a single feasible strategy.

For the initial model, each strategy set contains two feasible strategies: Lead (L) and Not Lead (NL). Let s_i' be Lead, and s_i'' be Not Lead. Therefore, Player 1 strategy set is:

$$\begin{aligned} S_1 &= \{s_1', s_1''\} \\ &= \{\text{Leads, Not Leads}\} \end{aligned}$$

Likewise, Player 2 strategy set is:

$$\begin{aligned} S_2 &= \{s_2', s_2''\} \\ &= \{\text{Leads, Not Leads}\} \end{aligned}$$

When each player chooses a feasible strategy to use, the specific combination of strategies chosen by each player is known as the strategy profile. Strategy profiles are denoted $s_{P_1 Action, P_2 Action}$. For example, if Player 1 chooses to Lead (s_1') and Player 2 chooses to Not Lead (s_2''), then this specific strategy profile is denoted as:

$$s_{L,NL} = \{s_1', s_2''\}$$

For the initial model, we will define all possible strategy profiles for Player 1 and Player 2:

Player 1 Leads and Player 2 Leads : $s_{L,L} = \{s_1', s_2'\}$
Player 1 Leads and Player 2 Not Leads : $s_{L,NL} = \{s_1', s_2''\}$
Player 1 Not Leads and Player 2 Leads : $s_{NL,L} = \{s_1'', s_2'\}$
Player 1 Not Leads and Player 2 Not Leads : $s_{NL,NL} = \{s_1'', s_2''\}$

1.2.C Payoffs

Each strategy profile will have certain outcomes for each player, these are known as payoffs. The payoffs for each player can be represented by a utility function, $u_i()$, where u takes on a strategy profile.

For the initial model, these numbers are arbitrarily assigned, however, they are meant to represent the real world scenarios of a championship race. For example, the payoffs for the strategy profile of a player who chooses to Lead and the other Not Lead is (1,2), respectively, which makes sense since the one who leads is exerting more force than the one who is not leading.

In that example, the strategy profile represents the real world scenario of one player leading while the other sits behind them. The player who is leading will have a small benefit for controlling the pace and a small benefit for "being in first", however, the player who leads is at a disadvantage because it is breaking the wind for the player behind it and also has to mentally work harder[6]. The player who Remains Same will be sitting behind the leader, and will gain the benefit of slipstreaming[2] as well as benefit from not having to mentally work as hard as the leader. Both players gain positive benefits for their actions, however, the one who is following behind the leader (in our model, the Not Lead action) will gain a bigger benefit than the player who leads.

For the initial model, we will define the payoffs from the utility function for Player 1 and Player 2 on all of the strategy profiles[3]:

$$u(s_{L,L}) = (1,1)$$
$$u(s_{L,NL}) = (1,2)$$
$$u(s_{NL,L}) = (2,1)$$
$$u(s_{NL,NL}) = (0,0)$$

[2]Slipstream is the region behind a runner which a wake of air occurs and causes reduced pressure and air resistance. If Runner B is behind Runner A, then Runner B will be in the Runner A's slipstream, which will require Runner B to use less energy to move at the same speed of Runner A.

[3]The first value in the payoff coordinates belongs to *Player One* and the second value belongs to *Player Two*

1.3 Normal-Form Model

A Normal-Form Model can be used in a static and simultaneous moved game, meaning player make a move at the same time. This is displayed as a matrix where the rows represent one of the player's strategies and the columns represent another player's strategies. If more than two players are involved, then multiple matrices are created to represent addition players' strategies. When looking at the initial model as a static, simultaneous move game, the following matrix can be used to represent the payoffs of each player:

		Player 2	
		Lead	Not Lead
Player 1	Lead	$(1, 1)$	$(1, 2)$
	Not Lead	$(2, 1)$	$(0, 0)$

Table 1: Payoff Matrix of the initial model (Normal-Form)

1.3.A Nash Equilibrium

Before analyzing a model (whether it is Normal-Form or Extensive-Form), it is important that common knowledge of the payoffs and rationality[4] of players are assumed. Common knowledge, a rather important concept in game theory, means that the information is known by all players, and all players know that all players know that information, and all player know that all player know that all players know that information, and so on.

When the payoffs of the initial model are assumed to be common knowledge, it means that: *Player One* knows all payoffs (its own and *Player Two*'s payoffs) and *Player Two* knows all payoffs (its own and *Player One*'s payoffs). It also means that, *Player One* knows that *Player Two* knows all payoffs and *Player Two* knows that *Player One* knows all payoffs. Even further, *Player One* knows that *Player Two* knows that *Player One* knows all the payoffs and *Player Two* knows that *Player One* knows that *Player Two* knows all the payoffs, and so on.

To evaluate the game in Normal-Form, Nash Equilibrium can be applied. A Nash Equilibrium is a strategy profile where each player's utility payoff is at least tied for its best response[7].Furthermore, John Nash proved, with the use of Kakutani's Theorem[5], that an equilibrium exists for any two player game[5].

For the initial model, a Nash Equilibrium can be found by underlining the best response (highest payoff) for each player given a specific strategy

[4]Rationality meaning players act in their best interest

[5]Shizuo Kakutani, PhD Osaka Univeristy. Fundamental theorem in Nash?s explanation of *Nash Equilibrium*

by the other player. For example, if *Player One* chooses to Lead, then *Player Two*'s best response is to Not Lead, and if *Player Two* chooses to Not Lead, then *Player One*'s best response (at least tied for best response) is to Lead. The following matrix represents all possible best responses for each player given the other player moves:

		Player 2	
		Lead	Not Lead
Player 1	Lead	$(1,1)$	$(\underline{1},\underline{2})$
	Not Lead	$(\underline{2},\underline{1})$	$(0,0)$

Table 2: Normal-Form with best responses underlined for each player

As it is seen in the matrix above, the Nash Equilibria for the initial model are:

$$\begin{aligned} NE &= \{(\text{Lead, Not Lead}), (\text{Not Lead, Lead})\} \\ &= \{(1,2),(2,1)\} \end{aligned}$$

1.4 Extensive-Form Model

An Extensive-Form can be used to represent a game theory model that is dynamic and sequential, meaning the agents take turns making moves and each move is a response to the previous. Extensive-Form is similar to a tree in graph theory. When using the Extensive-Form to represent a game, the nodes of the graph indicate a player. The edges of the graph indicate a specific strategy from the strategy set of the node (player) which the edge extends from. As one makes its way through the graph, each node it comes by represents a new round of the game which a decision ought to be made. Once one gets to the final node of the graph, the payoffs that each player will receive are shown. (these payoffs are identical to the payoffs from the Normal-Form game). A specific route, from the first node to a terminal node, is known as a path and represents a distinct strategy profile.

When showing a game with the Extensive-Form, the resulting graph is called the Game Tree[2]. For our initial model, the Extensive-Form can be represented by the following tree:

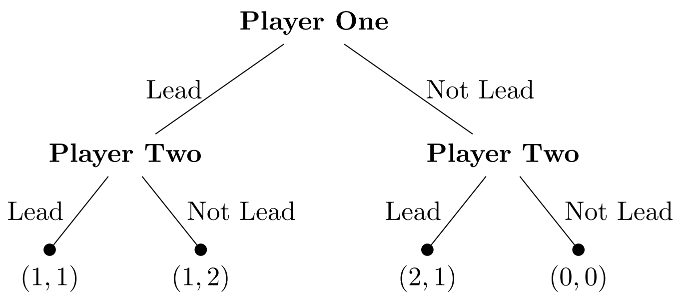

Figure 1: Game Tree of the initial model (Extensive-Form)

To fully understand the tree, lets walk through an example. *Player One* is the first node, which means that *Player One* has to choose a strategy: Lead or Not Lead. In this case, let's say *Player One* chooses Lead, s_1'. By moving through the tree, we land on a new node, *Player Two*. At this point *Player Two* has to choose a strategy: Lead or Not Lead. The logical decision for *Player Two* is to choose Not Lead, s_2'', since it leads to the greatest payoff for *Player Two*. Once again we move through the tree and stop at a terminal node, meaning the game is over and *Player One* receives a payoff of 1 and *Player Two* receives a payoff of 2. The path that was chosen represents the strategy profile $\{s_1', s_2''\}$.

Looking at the initial model through the Extensive-Form raises perhaps one concern - *Player One* has to make the first move. In the situation of a track race, we do not know which player will make the first move. It is too complicated to let the player who makes the first move be *Player One* since the terminal node payoffs will be changed depending on which player moves first.

To account for the unknown behavior of the players, we can introduce a new agent to the Extensive-Form tree, *Nature*. This *Nature* agent will be the starting point of the game and will give equal probability of either *Player One* or *Player Two* making the first move (in this case the probability of either player making the first move is 50, 50).

With *Nature* involved, the resulting Game Tree will look similar to the following graph:

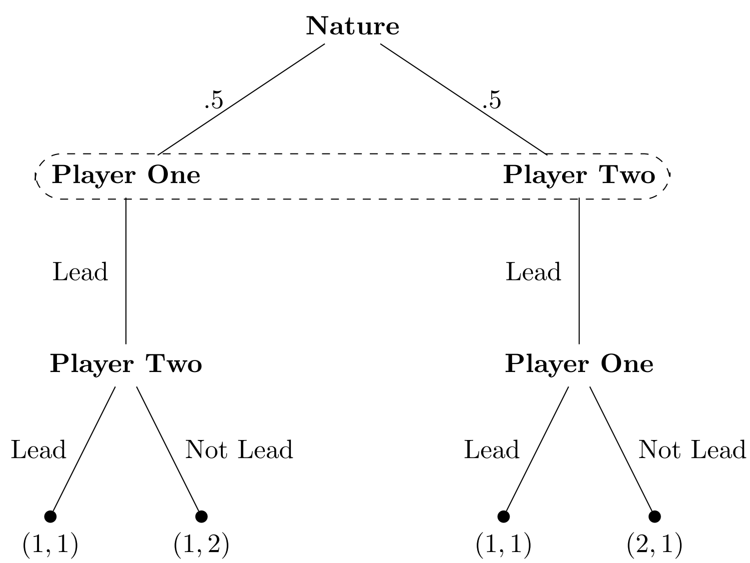

Figure 2: Game Tree of the initial model with Nature involved

1.4.A Backward Induction Equilibrium

To analyze a model in the Extensive-Form, once again common knowledge of the payoffs and rationality of players are assumed. To begin the evaluation, we will use a strategy known as Backward Induction. Backward Induction can be thought of as a process from starting at the end of the game tree and working backwards.

In the first game tree (Figure 1), which is a single round game, the desirable outcome for Player One is $(2, 1)$. If Player One chooses Not Lead, then Player Two will have potential outcomes, either 1 or 0. In this case, Player Two would choose the strategy that provides a payoff of 1 (if Player Two is rational). Since the payoff of 1 for Player Two corresponds to Player One receiving a payoff of 2, the strategy profile equilibrium via Backward Induction is for Player One to choose Not Lead and Player Two to choose to Lead.

$$BI = \{(\text{Not Lead}, \text{Lead})\} \quad = \{(2, 1)\}$$

Notice that the equilibrium via Backward Induction is not the exact same as the Nash Equilibria (it only contains one of the Nash Equilibria). This is completely acceptable in Game Theory. Backward Induction will not always be the same as Nash, and it can even yield equilibria that are not Nash equilibria.

2 Method

Now that a foundation of Game Theory modeling has been established, adding parameters will make the model realistic. However, adding parameters will also complicate the model. The objective is to create a realistic Payoff Function that represents the payoffs of each player depending on the strategy profile.

When developing a Payoff Function of a Game Theory model, a sensible payoff has to be determined. Since the purpose of Championship Style Racing is to preserve as much energy as possible and then use whatever energy you have left in the last lap so that you are in the lead by the finish[8], therefore, a sensible payoff function would represent the remaining energy a runner has.

To create a reasonable payoff function that uses the amount of energy used by a player, it is necessary to know what contributes to the amount of energy a runner uses while running. There are three main components that determines the amount of energy used while running: running form, running economy, and $VO2_{max}$[4].

The most realistic approach would be to create a payoff function that incorporates all of these components, however, this would complicate things beyond the scope of this paper. To ease the analysis of championship races on paper, a few assumptions have to be made: all players have the same running form and running economy. This allows focus on the $VO2_{max}$, which is the "measurement of the maximum amount of oxygen that an individual can utilize during maximal exercise"[3].

2.1 Maximum Energy Available

$VO2_{max}$ is measured in milliliters of oxygen utilized per kilogram of weight per min of exercise ($mL/kg/min$). A study done by the Department of Exercise Science at Syracuse University stated that for every $1mL$ of $VO2$ used is 20.1 joules of energy for every kg of weight[6]. Using Jack Daniel's VDot chart[6] it is possible to determine the total amount of energy an individual can use during a certain race given their $VO2_{max}$. The amount of maximum energy an individual can use is given by the following equation:

$$\Gamma = (V_i * w_i * 20.1) * t \tag{1}$$

where,

V_i represents *Player* i's $VO2_{max}$,
w_i represents *Player* i's weight,
t_i represents *Player* i's time racing,
and 20.1 is an energy constant for running

[6]The VDot chart shows the predicted times for different race distances (if a person is racing "all out") at specific $VO2_{max}$'s; see appendix for charts

To further understand the above formula, there is a *Player i* who is running a 1,500 meter race. If *Player i*'s $VO2_{max}$ is 70 and weight is $65kg$, then the amount of energy *Player i* uses is $91,455 \frac{Joules}{min}$. Since *Player i*'s $VO2_{max}$ is 70, it will take *Player i* 4 minutes to run 1,500 meters, meaning the maximum amount of energy *Player i* can use during a 1,500 meter race is $365,820\ J$:

$$\begin{aligned} P_i \text{ max energy} = \Gamma_i &= (V_i * w_i * 20.1) * t \\ &= (70 \frac{mL}{kg/min} * 65kg * 20.1) * t \\ &= (91,455 \frac{J}{min}) * 4min \\ &= 365,820J \end{aligned}$$

2.2 Payoff Function

Knowing a Player's maximum amount of energy available, the payoff function can be formulated. The payoff function will use the maximum amount of energy available to a Player and subtract the amount of energy used per round. The Player with the most energy left with 1 round (lap) to go is declared to be the winner. The payoff function is represented by the following equation:

$$\Gamma_i - \gamma_{1,i} - \gamma_{2,i} - \ldots - \gamma_{k-1,i}, \tag{2}$$

where,

Γ_i is *Player i*'s max. amount of energy,

$\gamma_{1,i}$ is the energy used in round 1 for *Player i*,

$\gamma_{2,i}$ is the energy used in round 2 for *Player i*,

$\gamma_{k-1,i}$ is the energy used in second to last round for *Player i*,

The amount of energy used in any distinct round by *Player i* is defined by:

$$\gamma_i = \begin{cases} [(v_i * w_i * 20.1) * t]\frac{1}{N} & \text{if } P_i \text{ chooses } \textbf{Lead} \\ 0.925[(v_i * w_i * 20.1) * t]\frac{N-1}{N} & \text{if } P_i \text{ chooses } \textbf{Not Lead} \end{cases} \tag{3}$$

where,

v_i is the $VO2$ used by P_i for a distinct round,

w_i is the weight of P_i,

t is the time the distinct round took, and

N is the number of players in the race

The formulation of a piecewise function for the amount of energy used for any given round is reasonable since the amount of energy used will be determined by the strategy chosen by the player.

The amount of energy a player uses during any given round is determined by their oxygen uptake ($VO2$) how much they weigh, the time (in minutes) they are running while at that specific oxygen uptake, and the energy constant for running, 20.1. However, when a player is not leading, they used 7.5% less energy since they have less air resistance[8] - this is where one of the differences come from between the strategies Lead and Not Lead. When a player does not lead, the energy used is multiplied by .925.

Also, when a player leads, the formula is multiplied by $\frac{1}{N}$ since majority of races are led by one player, meaning the probability that any given player has to lead is 1 out of the number of players in the race. When a player is not leading, the formula is multiplied by $\frac{N-1}{N}$ since only one player leads, the probability of not leading is one less than the number of players in the race divided by the number of player in the race.

3 Analysis

Now that the payoff function has been defined, analysis can be done to determine what maximizes the payoff function.

Based on the piecewise function, it is clear that for a certain number of players in the game, N:

$$0.925[(v_i * w_i * 20.1) * t]\frac{N-1}{N} > [(v_i * w_i * 20.1) * t]\frac{1}{N}$$

The first objective is to figure out what N satisfies the above inequality. Solving for N, we get:

$$\begin{aligned} 0.925[(v_i * w_i * 20.1) * t]\frac{N-1}{N} &> [(v_i * w_i * 20.1) * t]\frac{1}{N} \\ \frac{N-1}{N} &> 1.081 * \frac{1}{N} \\ N-1 &> 1.081 \\ N &> 2.081 \end{aligned}$$

Meaning, as long as the game consists of more than 2 players, then the optimal strategy to save energy is to Not Lead.

4 Discussion

The purpose of this article was to provide a basic, but necessary, foundation for creating a Game Theory Model, as well as provide a preliminary framework for future research. However, this article did have some limitations.

4.1 Limitations

One of the limitation is the assumptions of running form and running economy being fixed for all players. This would not be quite a realistic assumption since a track race will contain runners with varying form and economy. Without this assumption, the payoff function would have become too complex to evaluate on paper.

Another limitation is the use of Extensive-Form. It makes sense that Extensive-Form was used to describe the game, sense a track race is a sequential event (the runners react to different moves, they do not make moves simultaneously). However, Extensive-Form limits the number of "rounds." However, in a true track race, the runners can make moves at any point, not just at the beginning of each round (i.e. lap).

The final limitation is the lack of real-world data. Data is collect on runners at the beginning of each lap, however, this data does not tell whether a move was made and if a move was made, when it occurred. Also, there is no individual data on each runner's $VO2_{max}$. Without real-world data, it is hard to validate a theoretical model.

Do deal with these limitations, future research can be conducted with simulations. Simulations will eliminate the assumption of same running form and economy by allowing different inputs of these components for different agents in the game. Simulation will also eliminate the Extensive-Form by creating a continuous game rather than discrete, which will lead to a more realistic result. Lastly, Simulation will eliminate the need for real-world data since a multitude of simulations can be ran.

5 Conclusion

Modeling a championship race using a Game Theory framework proved to be complex. Looking at the initial model in Normal-Form and Extensive-Form, trivial solutions occurred ? the best strategy profile, based on Nash Equilibria and Backward Induction, is for Player One to Not Lead and Player Two to Lead.

With the assumptions that each runner has similar form and running economies, a payoff function was created that involved each runner's $V02_{max}$ and air resistance. It was found to maximize the payoff function, a runner prefers to Not Lead and stay behind the leader in a race. This will save energy (˜7.5%) due to the lack of air resistance, meaning the runner who does Not Lead will have more energy to finish first during the last round.

Also, with the introduction of Nature, which randomly assigned a runner to lead first, it was found that a runner who leads first is at a disadvantage. It was also found that with a large pool of runners racing, the better chance of a runner not having to lead the race, which will save the runner energy from being less likely to lead, which will lead to a greater chance of finishing first (compared to having to lead in a big field verse having to lead in a small field).

To better evaluate championship races in track by simulations can be used. This will allow for a broad evaluation with little assumptions made about the runners. Also, simulations will help with determining the optimal strategies, or optimal times to take the lead in a race to have a better chance at finishing first.

Appendices

A VDOT Charts

Table 1 VDOT values associated with times raced over some popular distances

VDOT	1500	Mile	3000	2-mile	5000	10,000	15,000	1/2 Mara	Marathon	VDOT
30	8:30	9:11	17:56	19:19	30:40	63:46	98:14	2:21:04	4:49:17	30
32	8:02	8:41	16:59	18:18	29:05	60:26	93:07	2:13:49	4:34:59	32
34	7:37	8:14	16:09	17:24	27:39	57:26	88:30	2:07:16	4:22:03	34
36	7:14	7:49	15:23	16:34	26:22	54:44	84:20	2:01:19	4:10:19	36
38	6:54	7:27	14:41	15:49	25:12	52:17	80:33	1:55:55	3:59:35	38
40	6:35	7:07	14:03	15:08	24:08	50:03	77:06	1:50:59	3:49:45	40
42	6:19	6:49	13:28	14:31	23:09	48:01	73:56	1:46:27	3:40:43	42
44	6:03	6:32	12:55	13:56	22:15	46:09	71:02	1:42:17	3:32:23	44
45	5:56	6:25	12:40	13:40	21:50	45:16	69:40	1:40:20	3:28:26	45
46	5:49	6:17	12:26	13:25	21:25	44:25	68:22	1:38:27	3:24:39	46
47	5:42	6:10	12:12	13:10	21:02	43:36	67:06	1:36:38	3:21:00	47
48	5:36	6:03	11:58	12:55	20:39	42:50	65:53	1:34:53	3:17:29	48
49	5:30	5:56	11:45	12:41	20:18	42:04	64:44	1:33:12	3:14:06	49
50	5:24	5:50	11:33	12:28	19:57	41:21	63:36	1:31:35	3:10:49	50
51	5:18	5:44	11:21	12:15	19:36	40:39	62:31	1:30:02	3:07:39	51
52	5:13	5:38	11:09	12:02	19:17	39:59	61:29	1:28:31	3:04:36	52
53	5:07	5:32	10:58	11:50	18:58	39:20	60:28	1:27:04	3:01:39	53
54	5:02	5:27	10:47	11:39	18:40	38:42	59:30	1:25:40	2:58:47	54
55	4:57	5:21	10:37	11:28	18:22	38:06	58:33	1:24:18	2:56:01	55
56	4:53	5:16	10:27	11:17	18:05	37:31	57:39	1:23:00	2:53:20	56
57	4:48	5:11	10:17	11:06	17:49	36:57	56:46	1:21:43	2:50:45	57
58	4:44	5:06	10:08	10:56	17:33	36:24	55:55	1:20:30	2:48:14	58
59	4:39	5:02	9:58	10:46	17:17	35:52	55:06	1:19:18	2:45:47	59
60	4:35	4:57	9:50	10:37	17:03	35:22	54:18	1:18:09	2:43:25	60

Figure 3: VDOT Chart for $VO2_{max}$'s from 30 to 60

VDOT	1500	Mile	3000	2-mile	5000	10,000	15,000	1/2 Mara	Marathon	VDOT
61	4:31	4:53	9:41	10:27	16:48	34:52	53:32	1:17:02	2:41:08	61
62	4:27	4:49	9:33	10:18	16:34	34:23	52:47	1:15:57	2:38:54	62
63	4:24	4:45	9:25	10:10	16:20	33:55	52:03	1:14:54	2:36:44	63
64	4:20	4:41	9:17	10:01	16:07	33:28	51:21	1:13:53	2:34:38	64
65	4:16	4:37	9:09	9:53	15:54	33:01	50:40	1:12:53	2:32:35	65
66	4:13	4:33	9:02	9:45	15:42	32:35	50:00	1:11:56	2:30:36	66
67	4:10	4:30	8:55	9:37	15:29	32:11	49:22	1:11:00	2:28:40	67
68	4:06	4:26	8:48	9:30	15:18	31:46	38:44	1:10:05	2:26:47	68
69	4:03	4:23	8:41	9:23	15:06	31:23	48:08	1:09:12	2:24:57	69
70	4:00	4:19	8:34	9:16	14:55	31:00	47:32	1:08:21	2:23:10	70
71	3:57	4:16	8:28	9:09	14:44	30:38	46:58	1:07:31	2:21:26	71
72	3:54	4:13	8:22	9:02	14:33	30:16	46:24	1:06:42	2:19:44	72
73	3:52	4:10	8:16	8:55	14:23	29:55	45:51	1:05:54	2:18:05	73
74	3:49	4:07	8:10	8:49	14:13	29:34	45:19	1:05:08	2:16:29	74
75	3:46	4:04	8:04	8:43	14:03	29:14	44:48	1:04:23	2:14:55	75
76	3:44	4:02	7:58	8:37	13:54	28:55	44:18	1:03:39	2:13:23	76
77	3:41+	3:58+	7:53	8:31	13:44	28:36	43:49	1:02:56	2:11:54	77
78	3:38.8	3:56.2	7:48	8:25	13:35	28:17	43:20	1:02:15	2:10:27	78
79	3:36.5	3:53.7	7:43	8:20	13:26	27:59	42:52	1:01:34	2:09:02	79
80	3:34.2	3:51.2	7:37.5	8:14.2	13:17.8	27:41.2	42:25	1:00:54	2:07:38	80
81	3:31.9	3:48.7	7:32.5	8:08.9	13:09.3	27:24	41:58	1:00:15	2:06:17	81
82	3:29.7	3:46.4	7:27.8	8:03.7	13:01.1	27:07	41:32	59:38	2:04:57	82
83	3:27.6	3:44.1	7:23.1	7:58.7	12:53.0	26:51	41:06	59:01	2:03:40	83
84	3:25.5	3:41.8	7:18.5	7:53.7	12:45.2	26:34	40:42	58:25	2:02:24	84
85	3:23.5	3:39.6	7:14.1	7:48.9	12:37.4	26:19	40:17	57:50	2:01:10	85

Figure 4: VDOT Chart for $VO2_{max}$'s from 61 to 85

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